UG Single Major Program 2023-24

PHYSICS

MINOR

FEBRUARY 14

Semester-II

Mechanics & Properties of Matter

Simplified Study Material

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S.Ch.V.P.M.R.Government Degree College

Ganapavaram, West Godavari District

II Semester
Physics Minor

Mechanics & Properties of Matter

Study Material
(English Medium)

Scan the below QR for Softcopy of the Study Material

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ANDHRA PRADESH STATE COUNCIL OF HIGHER EDUCATION

MINOR

Subject: Physics

w.e.f. AY 2023-24

COURSE STRUCTURE

Year	Semester	Course	Title of the Course	No. of Hrs /Week	No. of Credits
I	II	1	Mechanics and Properties of Matter	3	3
			Mechanics and Properties of Matter Practical Course	2	1
II	III	2	Optics	3	3
			Optics Practical Course	2	1
	IV	3	Electricity and Magnetism	3	3
			Electricity and Magnetism Practical Course	2	1
		4	Modern Physics	3	3
			Modern Physics Practical Course	2	1
III	V	5	Applications of Electricity & Electronics	3	3
			Applications of Electricity & Electronics Practical Course	2	1
		6	Electronic Instrumentation	3	3
			Electronic Instrumentation Practical Course	2	1

SEMESTER-II

COURSE 1: MECHANICS AND PROPERTIES OF MATTER

Theory Credits: 3 3hrs/week

COURSE OBJECTIVE:

The course on Mechanics and Properties of Matter aims to provide students with a fundamental understanding of the behaviour of physical systems, both in terms of mechanical motion and in terms of the properties of matter

LEARNING OUTCOMES:

1. Students will be able to understand and apply the concepts of scalar and vector fields, calculate the gradient of a scalar field, determine the divergence and curl of a vector field.
2. Students will be able to apply the laws of motion, solve equations of motion for variable mass systems
3. Students will be able to define a rigid body and comprehend rotational kinematic relations, derive equations of motion for rotating bodies, analyze the precession of a top and gyroscope, understand the precession of the equinoxes
4. Students will be able to define central forces and provide examples, understand the characteristics and conservative nature of central forces, derive equations of motion under central forces.
5. Students will be able to differentiate between Galilean relativity and the concept of absolute frames, comprehend the postulates of the special theory of relativity, apply Lorentz transformations, understand and solve problems

UNIT-I VECTOR ANALYSIS **9hrs**

Scalar and vector fields, gradient of a scalar field and its physical significance. Divergence and curl of a vector field with derivations and physical interpretation. Vector integration (line, surface and volume), Statement and proof of Gauss and Stokes theorems.

UNIT-II MECHANICS OF PARTICLES **9hrs**

Laws of motion, motion of variable mass system, Equation of motion of a rocket. Conservation of energy and momentum, Collisions in two and three dimensions, Concept of impact parameter, scattering cross-section, Rutherford scattering-derivation.

UNIT-III MECHANICS OF RIGID BODIES AND CONTINUOUS MEDIA **9hrs**

Definition of rigid body, rotational kinematic relations, equation of motion for a rotating body, Precession of a top, Gyroscope, Precession of the equinoxes. Elastic constants of isotropic solids and their relations, Poisson's ratio and expression for Poisson's ratio. Classification of beams, types of bending, point load, distributed load.

UNIT-IV CENTRAL FORCES **9hrs**

Central forces, definition and examples, characteristics of central forces, conservative nature of central forces, conservative force as a negative gradient of potential energy, equations of motion under a . Derivation of Kepler's laws. Motion of satellites

UNIT-V SPECIAL THEORY OF RELATIVITY **9hrs**

Galilean relativity, Absolute frames. Michelson-Morley experiment, The negative result. Postulates of special theory of relativity. Lorentz transformation, time dilation, length contraction, addition of velocities, mass-energy relation.

Unit-I
Vector Analysis

Scalar Field and Vector field

The value of a physical quantity changes from point to point in space. Every physical quantity is defined by a point function. The region in which the point function defines the physical quantity is known as a field.

Scalar field: If the value of a physical quantity at any point in a field is described by a scalar, then the field is called a scalar field. Scalar field is represented by $\emptyset(x, y, z)$. In a scalar field, the value of physical quantity changes only in magnitude from point to point.

Ex: Temperature, Electric Potential, Density etc…

Vector Field: If the value of a physical quantity at any point in a field is described by a vector, then the field is called a vector field. Vector field is represented by $\overline{A}(x, y, z)$. In a vector field, the value of physical quantity changes both in magnitude and direction from point to point.

Ex: Electric field intensity, Magnetic field intensity, velocity of a particle in a liquid etc..

Gradient of a Scalar Field

Gradient of a scalar field $\emptyset(x, y, z)$is defined as follows.

$$\text{Grad}\emptyset = \overline{i}\frac{\partial\emptyset}{\partial x} + \overline{j}\frac{\partial\emptyset}{\partial y} + \overline{k}\frac{\partial\emptyset}{\partial z} = \left(\overline{i}\frac{\partial}{\partial x} + \overline{j}\frac{\partial}{\partial y} + \overline{k}\frac{\partial}{\partial z}\right).\emptyset = \overline{\nabla}\emptyset$$

$$\nabla \equiv \overline{i}\frac{\partial}{\partial x} + \overline{j}\frac{\partial}{\partial y} + \overline{k}\frac{\partial}{\partial z} = \text{Vector Differential Operator}$$

➔ Gradient of a scalar field is a vector.

Physical significance of grad$\emptyset$:

$$d\emptyset = \frac{\partial\emptyset}{\partial x}dx + \frac{\partial\emptyset}{\partial y}dy + \frac{\partial\emptyset}{\partial z}dz$$

$$\overline{\nabla}\emptyset.\overline{dr} = \left(\overline{i}\frac{\partial\emptyset}{\partial x} + \overline{j}\frac{\partial\emptyset}{\partial y} + \overline{k}\frac{\partial\emptyset}{\partial z}\right).(dx\,\overline{i} + dy\,\overline{j} + dz\,\overline{k})$$

$$= \frac{\partial\emptyset}{\partial x}dx + \frac{\partial\emptyset}{\partial y}dy + \frac{\partial\emptyset}{\partial z}dz = d\emptyset = 0$$

$$\therefore \overline{\nabla}\emptyset.\overline{dr} = 0$$

$$\overline{\nabla}\emptyset \perp \overline{dr}$$

1. Direction of grad$\emptyset$ at any point gives the direction of maximum rate of increase of $\emptyset(x, y, z)$ at that point.
2. Magnitude of grad $\emptyset$ at any point gives the maximum rate of increase of $\emptyset(x, y, z)$ at that point.

Divergence of a Vector Field

Divergence of a vector field $\overline{A}$ is defined as the dot product of the vector field $\overline{A}$ with Deloperator $\overline{\nabla}$.

$$\text{Div}\overline{A} = \overline{\nabla}.\overline{A} = \left(\overline{i}\frac{\partial}{\partial x} + \overline{j}\frac{\partial}{\partial y} + \overline{k}\frac{\partial}{\partial z}\right).\left(A_x\overline{i} + A_y\overline{j} + A_z\overline{k}\right) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

➔ Divergence of a vector field is a scalar.

Physical Significance:

Divergence of a vector at any point gives the amount of vector flux emitted per unit volume at that point.

- If $\overline{v}$ is the velocity of a fluid, then $div\,\overline{v}$ gives that rate of flow of the fluid.
  - ✔ If$div\,\overline{v}$ is zero, then the liquid is incompressible.
  - ✔ If$div\,\overline{v}$ is positive, then the liquid is undergoing expansion.
  - ✔ If $div\,\overline{v}$ is negative, then the liquid is undergoing contraction.

Examples:

1. $div\,\overline{j} = \frac{\rho}{\epsilon_0}$

Here $\overline{j}$is the electric current density and ρis the charge density.

2. $div\,\overline{B} = 0$

Here $\overline{B}$ is the magnetic induction.

Curl of a vector field

Curl of a vector field is defined as the cross product of the vector with the del operator.

$$\text{Curl}\overline{A} = \overline{\nabla}\times\overline{A} = \left(\overline{i}\frac{\partial}{\partial x} + \overline{j}\frac{\partial}{\partial y} + \overline{k}\frac{\partial}{\partial z}\right)\times\left(A_x\overline{i} + A_y\overline{j} + A_z\overline{k}\right)$$

➔ Curl of a vector field is a vector.

Significance:

- Curl means rotation. Hence the curl of a vector field gives the rotation of the vector.
  - ✔ If the curl of a vector is zero, then the vector is called irrotational.
  - ✔ If the curl of a vector is non-zero, then the vector has rotation.
- Curl of a vector at any point gives the maximum line integral of the vector around a closed curve at that point.

Examples:

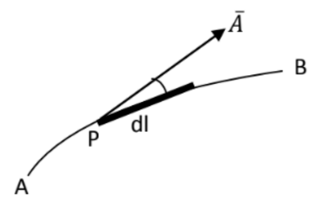
1. $Curl\overline{v} = 2\overline{\omega}$

   Here $\overline{v}$ is linear velocity and $\overline{\omega}$ is angular velocity

2. $Curl\overline{B} = \mu_0\overline{J}$

   Here $\overline{B}$ is the magnetic induction and $\overline{J}$ is the current density.

Line integral



Consider a curve AB as shown in figure. Consider a small element of length dl on the curve AB. Let be a vector at the point P. Let θ be the angle between the vectors $\overline{A}$ and the element $\overline{dl}$.

Then the line integral of the vector $\overline{A}$along the curve AB is given by

$$\int_A^B \overline{A}.\overline{dl} = \iint_A^B A\cos\cos\theta\, dl$$

If $\overline{A} = A_x\overline{i} + A_y\overline{j} + A_x\overline{k}$ and $\overline{dl} = dx\,\overline{i} + dy\,\overline{j} + dz\,\overline{k}$

Then $\int_A^B \overline{A}.\overline{dl} = \int_A^B \left(A_x dx + A_y dy + A_x dz\right)$

Example:

$$\int \overline{B}.\overline{dl} = \mu_0 I$$

Where $\overline{B}$ is the magnetic field and I is the electric current.

Surface Integral

Consider a surface S. Consider a small element of area ds on the surface S. Let $\overline{A}$ be a vector at the point 'P'. Let θ be the angle between the vectors $\overline{A}$ and the normal to the element $\overline{ds}$.

Then the surface integral of the vector $\overline{A}$over the surface S is given by

$$\iint \overline{A}.\overline{ds} = \iint A\cos\cos\theta\, ds$$

Example:

$$\iint \overline{B}.\overline{ds} = 0$$

Where $\overline{B}$ is the magnetic field.

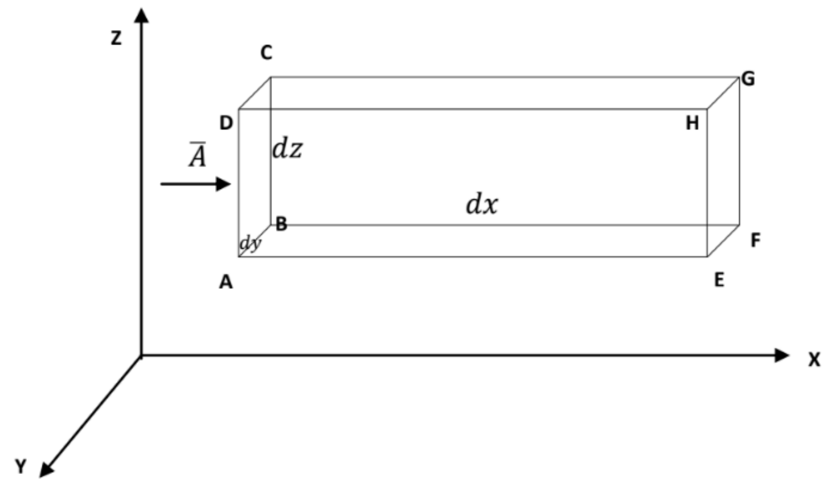
Gauss Divergence Theorem

Statement: Surface integral of a vector $\overline{A}$ over a closed surface S is equal to the volume integral of the divergence of the vector $\overline{A}$ over the volume V bounded by the surface S.

$$\iint_S \overline{A}.\,d\overline{S} = \iiint_V div\overline{A}\, dV$$

Proof:

Consider a volume V enclosed by a closed surface S. Let us divide the volume V into a large number of small volume elements . Let us assume that the volume element dV is in the shape of parallelepiped. Let dx, dy, dz be the lengths of parallelepiped along X, Y, Z axes. The vector $\overline{A}$ is acting along the face $ABCD$ as shown in figure.



Area of the face $ABCD = dy\,dz$

Hence the amount of flux entering the parallelepiped through the face $ABCD$ is given by

$$= A_x(P_1)\,dy\,dz$$

Similarly the amount of flux leaving the parallelepiped along the face $EFGH$ is given by

$$= A_x(P_2)\,dy\,dz$$

Since the length of the parallelepiped is very small

$$A_x(P_2) = A_x(P_1) + \frac{\partial A_x}{\partial x}\,dx$$

Hence the net amount of flux

$$= A_x(P_2)\,dy\,dz - A_x(P_1)\,dy\,dz$$

$$= \left(A_x(P_1) + \frac{\partial A_x}{\partial x}\,dx\right)dy\,dz - A_x(P_1)\,dy\,dz$$

$$= A_x(P_1)\,dy\,dz + \frac{\partial A_x}{\partial x}\,dx\,dy\,dz - A_x(P_1)\,dy\,dz$$

$$= \frac{\partial A_x}{\partial x}\,dx\,dy\,dz$$

Net amount of flux along X-axis is given by

$$= \frac{\partial A_x}{\partial x}\,dx\,dy\,dz$$

Similarly the net amount of flux along Y-axis is given by

$$= \frac{\partial A_y}{\partial y}\,dx\,dy\,dz$$

Similarly the net amount of flux along Z-axis is given by

$$= \frac{\partial A_z}{\partial z}\,dx\,dy\,dz$$

Hence the total amount of flux leaving the volume element dV is given by

$$= \frac{\partial A_x}{\partial x}\,dx\,dy\,dz + \frac{\partial A_y}{\partial y}\,dx\,dy\,dz + \frac{\partial A_z}{\partial z}\,dx\,dy\,dz$$

$$= \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)dx\,dy\,dz$$

$$= \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)dV$$

$$= div\overline{A}dV$$

$$\therefore \iint_S \overline{A}.\,d\overline{S} = div\overline{A}dV$$

Amount of flux leaving the total volume V is equal to the total flux from all the volume elements dV.

$$\therefore \iint_S \overline{A}.\,d\overline{S} = \iiint_V div\overline{A}\,dV$$

This is known as Gauss divergence theorem.

Stokes Theorem

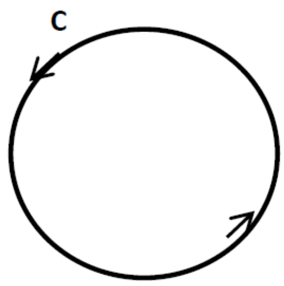
Statement: Line integral of a vector $\overline{A}$ around a closed curve C is equal to the surface integral of the curl of the vector $\overline{A}$ over the surface bounded by the curve C

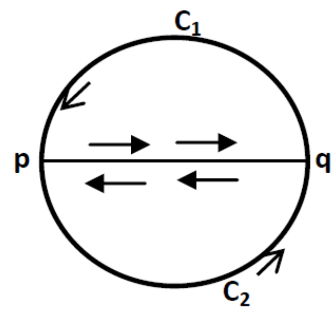
$$\oint_C \overline{A}.d\overline{r} = \iint_S Curl\,\overline{A}.\,d\overline{S}$$

Proof:

Line integral of the vector$\bar{A}$ around the closed curve C

$$L = \oint_C \bar{A}.\bar{dr}$$

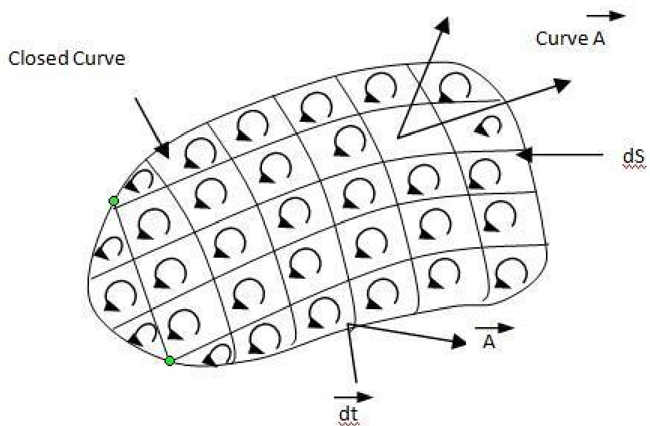




Let us divided the curve C in to two parts C_1, C_2 by the line pq. Hence the area enclosed by the curve C is divided into two parts.Let L_1 be the line integral of the vector around the curve $C_1$and L_2 be the line integral of the vector around the curve C_2.

$$L = L_1 + L_2$$

Since the line integral around the curve C_1 is from p to q and the line integral around the curve C_2 is from q to p. Hence the line integral along the line pq need not be considered.



Let us divide the area enclosed by the curve C in to a large number of small area elements $dS_1, dS_2, dS_3,$ .... These area elements $dS_1, dS_2, dS_3,$ ....are enclosed by the curves C_1, C_2, C_3.........

$$\oint_C \bar{A}.\bar{dr} = \sum \oint_{C_n} \bar{A}.\bar{dr}$$

We know that the line integral of a vector $\bar{A}$ at a point is equal to the maximum line integral of the vector around the curve at that point. Hence the line integral of the vector $\bar{A}$ around the curve enclosing the element dS_1 is given by

$$\oint_{C_1} \bar{A}.\bar{dr} = (\nabla \times \bar{A}).\, d\bar{S_1} \quad ....\mathbf{1}$$

The total value of line integral is obtained by adding all the integrals on the left hand side of equation

$$\oint_{C_1} \bar{A}.\bar{dr} + \oint_{C_2} \bar{A}.\bar{dr} + \oint_{C_3} \bar{A}.\bar{dr} + \text{..........} = \oint_C \bar{A}.\bar{dr} \quad \text{....................}\mathbf{2}$$

Similarly adding the integrals on the right hand side of the equation we get the total value of the surface integral $(\nabla \times \bar{A}).\, d\bar{S}$over all the surfaces $dS_1, dS_2, dS_3,$ ....

$$\iint_S (\nabla \times \bar{A}).\, d\bar{S} \quad \text{............}\mathbf{3}$$

From equations 2 and 3

$$\oint_C \bar{A}.\bar{dr} = \iint_S (\nabla \times \bar{A}).\, d\bar{S}$$

This is called Stokes Theorem.

UNIT-II
Mechanics of particles
Newton's laws of motion

Newton's first law:

Every body continues to be in a state of rest or uniform motion unless it is acted by an external force.

Newton's second law:

The net external force acting on a body is directly proportional to the rate of change of momentum.

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma$$
$$F = ma$$

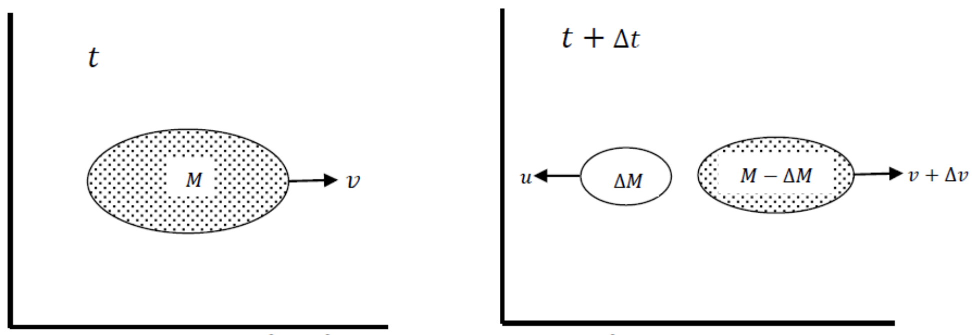
Newton's third law:

Every action has an equal and opposite reaction.

$$F_{12} = -F_{21}$$

Equation of motion of a system of variable mass

If the mass of a system changes with time without remaining constant, such a system is known as a system of variable mass. Motion of the rocket is an example of a system of variable mass. When the fuel inside the combustion chamber of a rocket is burnt, the burnt gases are ejected from the rocket in the form of a gas jet with high velocity in backward direction. As a result, the mass of the rocket decreases gradually and its velocity increases.



Consider a system of mass M moving with velocity v as shown in figure. After a time Δt, a mass ΔM is ejected from the system with velocity u. As a result, mass of the system is reduced to $(M - \Delta M)$and its velocity increased to $(v + \Delta v)$.

Initial momentum $P_i = Mv$

Final momentum $P_f = (M - \Delta M)(v + \Delta v) + \Delta M u$

Change in momentum $\Delta P = P_f - P_i = (M - \Delta M)(v + \Delta v) + \Delta M u - Mv$

According to Newton's second law

$$F_{ext} = \frac{dP}{dt} = \frac{\Delta P}{\Delta t} = \frac{(M-\Delta M)(v+\Delta v)+\Delta M u - Mv}{\Delta t} = \frac{Mv + M\Delta v - v\Delta M - \Delta v\Delta M + \Delta M u - Mv}{\Delta t}$$

$$F_{ext} = M\frac{\Delta v}{\Delta t} - v\frac{\Delta M}{\Delta t} - \Delta v\frac{\Delta M}{\Delta t} + u\frac{\Delta M}{\Delta t}$$

$$If\ \Delta t \to 0,\ then\ \frac{\Delta v}{\Delta t} = \frac{dv}{dt},\quad \frac{\Delta M}{\Delta t} = -\frac{dM}{dt},\ \Delta v \approx 0$$

$$F_{ext} = M\frac{dv}{dt} + v\frac{dM}{dt} - u\frac{\Delta M}{\Delta t} = \frac{d}{dt}(Mv) - u\frac{\Delta M}{\Delta t}$$

$$\boxed{F_{ext} = \frac{d}{dt}(Mv) - u\frac{\Delta M}{\Delta t}}$$

The above equation represents the equation of motion of a system of variable mass.

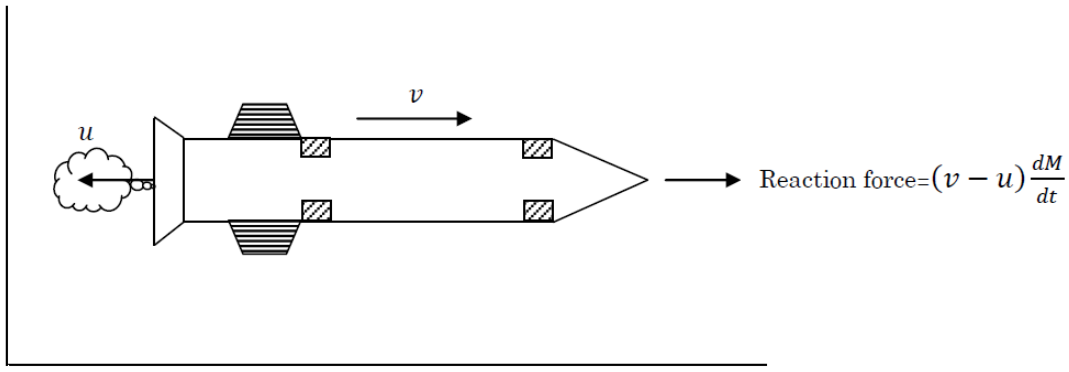
$$F_{ext} = M\frac{dv}{dt} + v\frac{dM}{dt} - u\frac{\Delta M}{\Delta t}$$
$$M\frac{dv}{dt} = F_{ext} + u\frac{\Delta M}{\Delta t} - v\frac{dM}{dt}$$
$$M\frac{dv}{dt} = F_{ext} + (u - v)\frac{dM}{dt}$$

Reaction force or thrust acting on the rocket is given by

$$F_{reaction} = (u - v)\frac{dM}{dt}$$

$$\boxed{M\frac{dv}{dt} = F_{ext} + F_{reaction}}$$

Expression for final velocity of a rocket



Motion of the rocket is an example of a system of variable mass. When the fuel in the combustion chamber of a rocket is burnt, pressure inside the chamber increases. Hence the hot gases inside the combustion chamber are ejected from the rocket in the form of a gas jet with high velocity in backward direction through a nozzle. Hence mass of the rocket decreases gradually due to the ejected gases and its velocity increases.

Consider a rocket of mass M moving with a velocity v at time t as shown in figure. After a time Δt, fuel of mass dM is ejected from the rocket with a velocity u in the form of a gas jet. Hence the velocity of the gas jet relative to the laboratory frame of reference is $(v - u)$.

Relative velocity $v_{relative} = v - u$

Reaction force on the rocket

$$F_{reaction} = (v - u)\frac{dM}{dt}$$

External force on the rocket

$$F_{ext} =- Mg$$

Hence the resultant force on the rocket in upward direction

$$F = F_{reaction} + F_{ext}$$

$$F = (v - u)\frac{dM}{dt} - Mg$$

According to Newton's second law

$$F = \frac{dP}{dt} = \frac{d}{dt}(Mv)$$

$$\frac{d}{dt}(Mv) = (v - u)\frac{dM}{dt} - Mg$$

$$M\frac{dv}{dt} + v\frac{dM}{dt} = v\frac{dM}{dt} - u\frac{dM}{dt} - Mg$$

$$M\frac{dv}{dt} =- u\frac{dM}{dt} - Mg$$

$$\frac{dv}{dt} =- \frac{u}{M}\frac{dM}{dt} - g$$

$$dv =- u\frac{dM}{M} - gdt$$

Let v_0, v be the initial final velocities and M_0, M be the initial and final masses of the rocket. Integrating the above equation on both sides

$$\int_{v_0}^{v} dv =- u\int_{M_0}^{M}\frac{dM}{M} - g\int_0^t dt$$

$$(v)_{v_0}^{v} =- u(\log M)_{M_0}^{M} - g(t)_0^t$$

$$v - v_0 =- u\left(\log M - \log M_0\right) - gt$$

$$v - v_0 =- u\log\frac{M}{M_0} - gt$$

$$v - v_0 = u\log\frac{M_0}{M} - gt$$

$$\boxed{v = v_0 + u\log\frac{M_0}{M} - gt}$$

The above expression represents the final velocity of the rocket.

Case(i):

Ignoring gravity, $g \approx 0$

$$v = v_0 + u\log\frac{M_0}{M}$$

Case(ii):

If the initial velocity of the rocket is zeroi.e $v_0 = 0$

$$v = u\log\frac{M_0}{M}$$

Two dimensional elastic collision

Let a particle of mass $m_1$moving with velocity $u_1$collides with another particle of mass m_2 at rest. After collision, mass m_1 is scattered at an angle $\theta_1$with the original direction. Similarly mass m_2 is scattered through an angle θ_2. Let v_1, v_2be the velocities of the two masses after collision.

Applying law of conservation of linear momentum along $X - axis$,

$$m_1u_1 + 0 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

$$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{1}$$

Similarly, applying law of conservation of linear momentum along $Y - axis$

$$0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2$$

$$m_1v_1\sin\theta_1 = m_2v_2\sin\theta_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{2}$$

According to law of conservation of kinetic energy,

$$\frac{1}{2}m_1u_1^2 + 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{3}$$

Let $m_1 = m_2$to solve the above equations.

From eqn **1**

$$u_1 = v_1\cos\theta_1 + v_2\cos\theta_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{4}$$

From eqn **2**

$$v_1\sin\theta_1 = v_2\sin\theta_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{5}$$

From eqn **3**

$$u_1^2 = v_1^2 + v_2^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{6}$$

From eqn **4**

$$u_1 - v_1\cos\theta_1 = v_2\cos\theta_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{7}$$

Squaring on both sides,

$$\left(u_1 - v_1\cos\theta_1\right)^2 = v_2^2\cos^2\theta_2$$

$$u_1^2 + v_1^2\cos^2\theta_1 - 2u_1v_1\cos\theta_1 = v_2^2\cos^2\theta_2$$

From eqn **5**

$$v_1\sin\theta_1 = v_2\sin\theta_2$$

Squaring on both sides

$$v_1^2\sin^2\theta_1 = v_2^2\sin^2\theta_2 \quad \ldots\ldots\ldots \mathbf{8}$$

Adding eqns **7** and **8**

$$u_1^2 + v_1^2 - 2u_1v_1\cos\theta_1 = v_2^2$$

From eqn **6**

$$u_1^2 = v_1^2 + v_2^2$$

$$v_1^2 + v_2^2 + v_1^2 - 2u_1v_1\cos\theta_1 = v_2^2$$

$$2v_1^2 - 2u_1v_1\cos\theta_1 = 0$$

$$v_1 - u_1 \cos\theta_1 = 0$$

$$v_1 = u_1 \cos\theta_1 \quad ....\mathbf{9}$$

From eqn **6**

$$v_2^2 = u_1^2 - u_1^2\cos^2\theta_1 = u_1^2\sin^2\theta_1$$

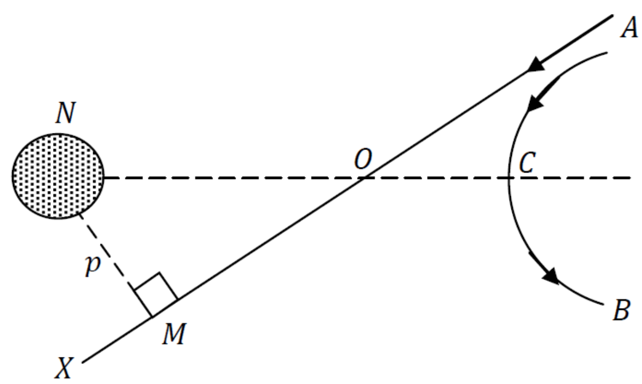
$$v_2 = u_1 \sin\theta_1 \quad ....\mathbf{10}$$

From eqns **9, 10** it is clear that v_1, v_2 are normal components of u_1.

$$\theta_1 + \theta_2 = 90^0$$

Hence in a perfectly elastic collision between two particles of equal masses, when one particle is initially at rest, the two particles always move off right angles to each other after collision.

Impact Parameter

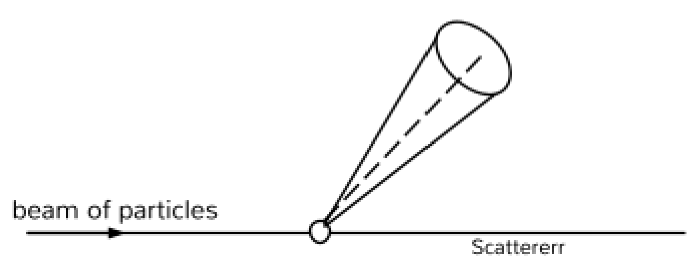


Consider an alpha particle of mass m and charge $+2e$ moving towards a nucleus of charge $+Ze$ in AX direction. Alpha particle follows a hyperbolic path ACB instead of a straight path AX due to Coulomb's repulsion of the nucleus. p is the perpendicular distance from nucleus N to the initial direction of the alpha particle. This is known as the Impact parameter. Hence Impact parameter can be defined as follows.

- Impact parameter (p) is defined as the perpendicular distance from the nucleus to the initial direction of the projected alpha particle.

If Impact parameter $p = 0$, then the collision is known as direct collision. In this case, the scattering angle $\emptyset = 0$.

Collision cross-section (or) Scattering cross-section

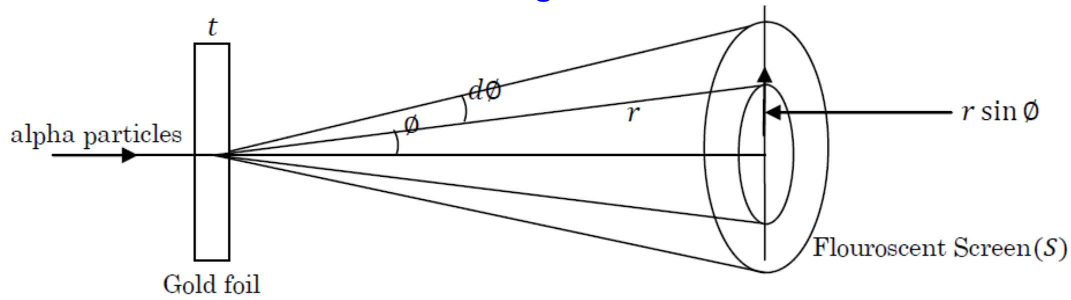


When alpha particles are incident on a thin gold foil, they are scattered in different directions. Let N be the incident intensity of the alpha particles. Let dN be the number of alpha particles scattered in to solid angle $d\omega$. The ratio of number of alpha particles scattered in to solid angle $d\omega$ and the incident intensity is known as the Impact parameter.

$$\text{Scattering cross-section}(\sigma) = \frac{Number\ of\ alpha\ particles\ scattered\ into\ solid\ angle\ d\omega}{Incident\ intensity}$$

$$\sigma = \frac{dN}{N}$$

Rutherford's Scattering Cross-section



Consider a narrow beam of alpha particles incident normally on a gold foil as shown in figure. Alpha particles are scattered in different directions due to coulomb's repulsive force of the nucleus. A fluorescent screen (S) is used to detect the scattered alpha particles. Let t be the thickness of the gold foil and N be the number of atoms per unit volume. Let Q be the number of alpha particles incident on the gold foil per unit area. Any alpha particle which comes within a distance of impact parameter (p) from the nucleus will be scattered through an angle Ø. Hence in order to calculate the number of alpha particles scattered through an angle Ø. Let us imagine a circle of radius equal to the impact parameter around each nucleus. Total area of all such circles is $\pi p^2 nt$.

- Probable number of alpha particles which can come within a distance p from the nucleus

$$= \pi p^2 ntQ.$$

- Number of alpha particles having impact parameter between p and $p + dp$

$$= d\left(\pi p^2 ntQ\right) = 2\pi pntQdp$$

- Hence the number of alpha particles having scattered through an angle between Ø and $Ø + dØ$

$$= 2\pi pntQdp$$

$$\text{Scattering cross-section}(\sigma) = \frac{\text{Number of alpha particles scattered in to solid angle } d\omega}{\text{Incident intensity}}$$

Solid angle between Ø and $Ø + dØ = 2\pi \sin Ø\, dØ$

- Hence number of alpha particles scattered in to solid angle $d\omega$

$$= \sigma I\, d\omega = \sigma I\, 2\pi \sin Ø\, dØ$$

This value should be equal to the number of alpha particles having impact parameter between p and $p + dp$.

- Number of alpha particles having impact parameter between p and $p + dp$

$$= 2\pi p\, dp$$

$$\text{Number of incident alpha particles} = 2\pi p\, dp.I$$

$$\therefore \ \sigma I\, 2\pi \sin Ø\, dØ = -\, 2\pi p\, dp.I$$

$$\sigma = \frac{-2\pi p\, dp.I}{2\pi \sin Ø\, dØ.I} = -\frac{p\, dp}{\sin Ø\, dØ}$$

$$\sigma = -\frac{p\, dp}{\sin Ø\, dØ}$$

$$p = \frac{Ze^2}{2\pi\epsilon_0 m v_0^2} \cot\frac{Ø}{2}$$

$$dp = \frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\left(-\frac{1}{2} cosec^2 \frac{Ø}{2}\, dØ\right)$$

$$\sigma = \frac{\left(\frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\right)^2 \cot\frac{Ø}{2}\left(-\frac{1}{2}cosec^2\frac{Ø}{2}\, dØ\right)}{\sin Ø\, dØ} = \frac{\left(\frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\right)^2 \cot\frac{Ø}{2}\left(-\frac{1}{2}cosec^2\frac{Ø}{2}\, dØ\right)}{2\sin\frac{Ø}{2}\cos\frac{Ø}{2}\, dØ}$$

$$\boxed{\sigma = \frac{Z^2 e^4}{16\pi^2 \epsilon_0^2 m^2 v_0^4 \sin^4\frac{Ø}{2}}}$$

This is known as Rutherford' Scattering cross-section.

Rutherford's scattering formula:

Number of alpha particles scattered through an angle between Ø and $Ø + dØ$

$$= 2\pi pntQdp$$

Substituting the values of p and $p + dp$ in the above equation,
Number of alpha particles scattered through angle between $\emptyset$ and $\emptyset + d\emptyset$

$$= 2\pi ntQ\left(\frac{Ze^2}{2\pi\epsilon_0 mv_0^2}\cot\frac{\emptyset}{2}\right)\left[\frac{Ze^2}{2\pi\epsilon_0 mv_0^2}\left(-\frac{1}{2}cosec^2\frac{\emptyset}{2}\,d\emptyset\right)\right]$$

These particles strike the screen (S) in a circular annulus of area dA

$$dA = 2\pi r\sin\emptyset\, rd\emptyset = 2\pi r^2\sin\emptyset\, d\emptyset = 4\pi r^2\sin\frac{\emptyset}{2}\cos\frac{\emptyset}{2}\, d\emptyset$$

Number of alpha particles incident on the screen per unit area

$$N = \frac{2\pi ntQ\left(\frac{Ze^2}{2\pi\epsilon_0 mv_0^2}\cot\frac{\emptyset}{2}\right)\left[\frac{Ze^2}{2\pi\epsilon_0 mv_0^2}\left(-\frac{1}{2}cosec^2\frac{\emptyset}{2}\,d\emptyset\right)\right]}{4\pi r^2\sin\frac{\emptyset}{2}\cos\frac{\emptyset}{2}\, d\emptyset}$$

$$N = \frac{QntZ^2e^4}{16\pi^2\epsilon_0^2 r^2 m^2 v_0^4\sin^4\frac{\emptyset}{2}}$$

This is known as Rutherford's scattering formula.
Hence from the above equation, it is clear that the number of alpha particles scattered per unit area is

- Inversely proportional to $\sin^4\frac{\emptyset}{2}$
- Directly proportional to the thickness of gold foil $'t'$
- Directly proportional to the square of the atomic number $'Z'$ of the scatterer
- Inversely proportional to the square of the kinetic energy of the alpha particle.

Unit-III
Mechanics of Continuous media

Elasticity:

When external deformation forces are applied on a body, its size and shape change. The property of a body to regain its original shape and size when the external forces are removed is known as Elasticity.

Elastic Moduli of Isotropic Solids

- Young's Modulus (Y)
- Bulk Modulus (K)
- Rigidity Modulus (η)
- Poisson's Ratio (σ)

Young's Modulus (Y)**:**

- Ratio of Longitudinal Stress to Longitudinal Strain is called Young's Modulus.

$$\text{Young's Modulus } (Y) = \frac{Longitudinal\ Stress}{Longitudinal\ Strain}$$

Consider a rod of length l and cross sectional area a .Let δl be the change in length of the rod when a force F is applied along its length.

$$\text{Longitudinal Stress} = \frac{F}{a}$$

$$\text{Longitudinal Strain} = \frac{\delta l}{l}$$

$$\text{Young's Modulus } (Y) = \frac{\frac{F}{a}}{\frac{\delta l}{l}} = \frac{Fl}{a\delta l}$$

Bulk Modulus(K)**:**

- Ratio of Normal Stress to Volume Strain is known as Bulk modulus.

$$\text{Bulk Modulus}(K) = \frac{Normal\ Stress}{Volume\ Strain}$$

Consider a body of Volume V and Cross sectional area a. Let δV be the change in volume when a normal force F is applied over the entire surface of the body.

$$\text{Normal Stress} = \frac{F}{a}$$

$$\text{Volume Strain} = \frac{\delta V}{V}$$

$$\text{Bulk Modulus}(K) = \frac{\frac{F}{a}}{\frac{\delta V}{V}} = \frac{FV}{a\delta V}$$

Rigidity Modulus (η)**:**

- Ratio of Shearing Stress to Shearing Strain is called Rigidity modulus.

$$\text{Rigidity Modulus } (\eta) = \frac{Shearing\ Stress}{Shearing\ Strain}$$

Consider a body of Cross sectional area a. Let θ be the angle of deformation when a shearing force F is applied to it.

$$\text{Rigidity Modulus } (\eta) = \frac{\frac{F}{a}}{\theta}$$

Poisson's Ratio (σ)**:**

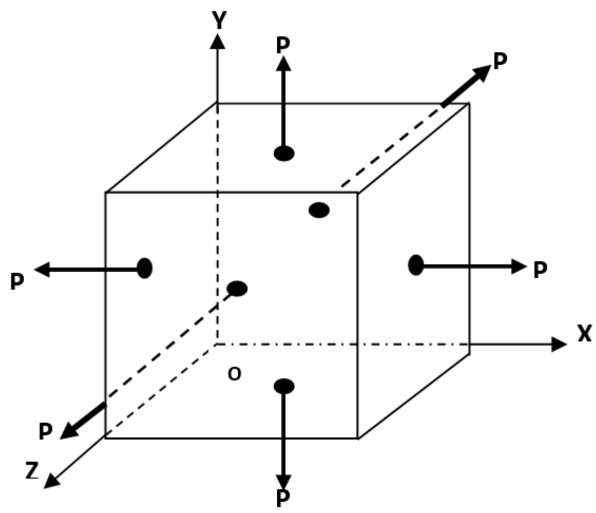
- Ratio of Transverse Strain to Longitudinal Strain is called Poisson's ratio.

$$\text{Poisson's Ratio } (\sigma) = \frac{Transverse\ Strain}{Longitudinal\ Strain}$$

Consider a wire of length l and diameter . Let δl be the increase in length and δD be the decrease in diameter when a force F is applied to it.

$$\text{Poisson's Ratio } (\sigma) = \frac{\frac{\delta D}{D}}{\frac{\delta l}{l}} = \frac{l\,\delta D}{D\,\delta l}$$

Relation between Elastic Moduli or Elastic Constants



Consider a unit cube as shown in figure. Let the faces of the cube are parallel to the coordinate axes X, Y, Z. Let a Normal Stress P is acting on all the six faces in an outward direction.

Young's Modulus $(Y)= \frac{Longitudinal\ Stress}{LOngitudinal\ Strain}$

$$Longitudinal\ Strain = \frac{Longitudinal\ Stress}{Young's\ Modulus} = \frac{\frac{P}{1}}{Y} = \frac{P}{Y}$$

Expansion along X axis= $\frac{P}{Y}$

Poisson's Ratio $(\sigma)=\frac{Transverse\ Strain}{Longitudinal\ Strain}$

$$Transverse\ Strain = Poisson's\ ratio \times Longitudinal\ Strain = \sigma\ \frac{P}{Y}$$

Compression along Y, Z axes $= \sigma\ \frac{P}{Y}$

Hence due to the Normal Stress P along X-axis

Expansion along X axis $= \frac{P}{Y}$

Compression along Y, Z axes $= \sigma\ \frac{P}{Y}$

Stress	**Strain**		
	X axis	**Y axis**	**Z axis**
$Stress\ along\ X - axis$	$\frac{P}{Y}$	$-\ \sigma\ \frac{P}{Y}$	$-\ \sigma\ \frac{P}{Y}$
$Stress\ along\ Y - axis$	$-\ \sigma\ \frac{P}{Y}$	$\frac{P}{Y}$	$-\ \sigma\ \frac{P}{Y}$
$Stress\ along\ Z - axis$	$-\ \sigma\ \frac{P}{Y}$	$-\ \sigma\ \frac{P}{Y}$	$\frac{P}{Y}$

Relation between Bulk Modulus and Young's Modulus:

Let e_x, e_y, e_z be the expansions along X, Y, Z axes due all the normal forces.

$$e_x = \frac{P}{Y} - \sigma\ \frac{P}{Y} - \ \sigma\ \frac{P}{Y} = \frac{P}{Y} - 2\sigma\ \frac{P}{Y} = \frac{P}{Y}(1 - 2\sigma)$$

$$e_x = \frac{P}{Y}(1 - 2\sigma)$$

$$e_y = \frac{P}{Y}(1 - 2\sigma)$$

$$e_z = \frac{P}{Y}(1 - 2\sigma)$$

Hence Longitudinal Strain in any direction= $\frac{P}{Y}(1 - 2\sigma)$

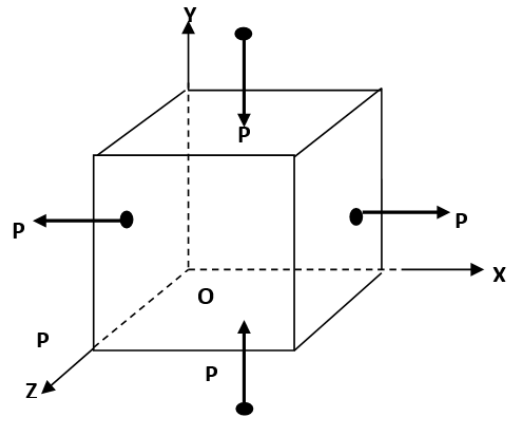
Volume Strain $= 3\times\ Longitudinal\ Strain = 3\frac{P}{Y}(1 - 2\sigma)$

Bulk Modulus$(K) = \frac{Normal\ Stress}{Volume\ Strain}$

$$K = \frac{P}{3\frac{P}{Y}(1-2\sigma)} = \frac{Y}{3(1-2\sigma)}$$

$$\boxed{Y = 3K(1 - 2\sigma)}$$

Relation between Rigidity Modulus and Poisson's ratio:



Consider a unit cube as shown in figure. Let an expansive stress P is acting along X axis and a compressive stress P is acting along Y axis on the unit cube.

Let e_x, e_y, e_z be the expansions along X, Y, Z axes due all the normal forces.

$$e_x = \frac{P}{Y} + \sigma\frac{P}{Y} = \frac{P}{Y}(1+\sigma)$$

$$e_y = -\frac{P}{Y} - \sigma\frac{P}{Y} = -\frac{P}{Y}(1+\sigma)$$

$$e_z = -\sigma\frac{P}{Y} + \sigma\frac{P}{Y} = 0$$

Hence equal amounts of expansive and compressive strains are produced along X, Y axes. These two mutually perpendicular expansive and compressive strains produce a shearing strain.

Shearing Strain $\theta = 3\times Longitudinal\ Strain = 2\frac{P}{Y}(1+\sigma)$

Rigidity Modulus $(\eta) = \frac{Shearing\ Stress}{Shearing\ Strain} = \frac{P}{\theta} = \frac{P}{2\frac{P}{Y}(1+\sigma)} = \frac{Y}{2(1+\sigma)}$

$$\eta = \frac{Y}{2(1+\sigma)}$$

$$Y = 2\eta(1+\sigma)$$

Relation between Elastic Moduli

$$Y = 3K(1-2\sigma) \Rightarrow \frac{Y}{3K} = 1-2\sigma$$

$$Y = 2\eta(1+\sigma) \Rightarrow \frac{Y}{\eta} = 2(1+\sigma)$$

Adding these two equations,

$$\frac{Y}{3K} + \frac{Y}{2\eta} = 3$$

$$Y = \frac{9\eta K}{3K+\eta}$$

$$Y = 3K(1-2\sigma) \qquad Y = 2\eta(1+\sigma)$$

$$3K(1-2\sigma) = 2\eta(1+\sigma)$$

$$3K - 2\eta = 2\eta\sigma + 6K\sigma$$

$$3K - 2\eta = \sigma(2\eta + 6K)$$

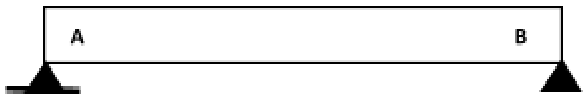
$$\sigma = \frac{3K-2\eta}{6K+2\eta}$$

Types of Beams

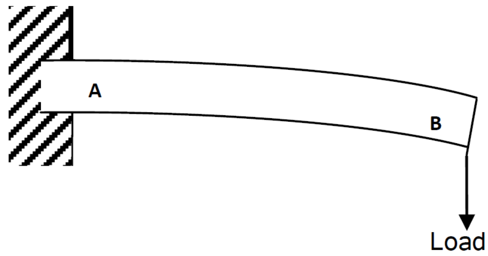
A structure designed to support loads applied perpendicular to its axis is known as a beam. Beams are of three types.

- Simple beam
- Cantilever beam
- Simple beam with overhang

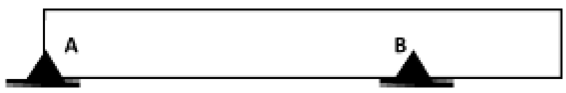
1. **Simple beam:** A beam supported at the two ends is known as a simple beam. One end of this beam is supported by a knife edge while the other end is supported by a roller. Simple beam is shown in the figure.



2. **Cantilever beam:** A beam in which one end is fixed and placed horizontally and weights are suspended at the other end is known as a cantilever beam. Cantilever beam is shown in figure.



3. **Simple beam with overhang:** A beam in which one end is extended in the form of a cantilever beyond its support is known as overhanging beam. Overhanging beam is shown in figure.



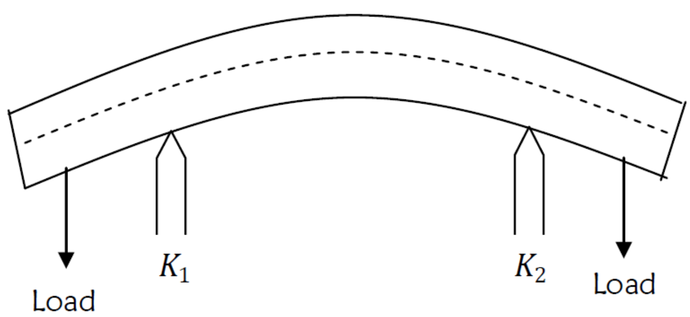
Types of bending

Bending of beams is of two types.

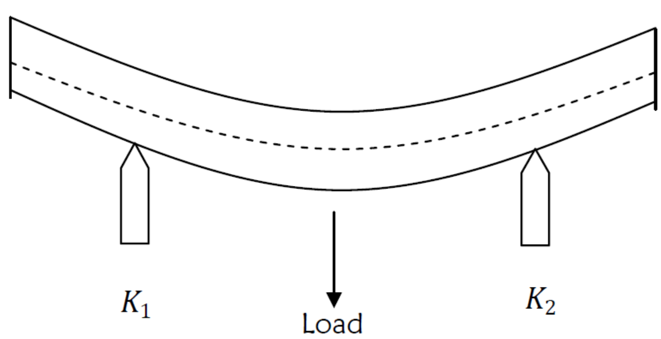
- **Uniform bending**
- **Non-uniform bending**

Radius of curvature of the neutral axis of a beam is known as the radius R of the beam.

Uniform bending: If the value of radius R is the same for all points of neutral axis, the bending is called uniform bending. The bending of a beam which is symmetrically supported on two knife edges and equally loaded at the two ends is an example for uniform bending. Uniform bending is shown in figure.



Non-uniform bending: If the value of radius R is different for different points of the neutral axis, the bending is called non-uniform bending. The bending of a beam which is symmetrically supported on two knife edges and loaded at the middle is an example for non-uniform bending. Non-uniform bending is shown in figure.



Unit-III
Mechanics of Rigidbodies
Euler's equations (or) Equations of motion of a rigid body

Equation of motion of a rigid body in space coordinate system

$$\vec{\tau}_{space} = \left(\frac{d\vec{L}}{dt}\right)_{space} \quad ....1$$

The rotation of a rigid body can also be studied by a coordinate system fixed in the rigid body.This is known as body coordinate system.

$$\vec{L}_{body} = \left(\overleftrightarrow{I}\vec{\omega}\right)_{body} \quad ....2$$

We can transform the equations of motion of a rigid body from body coordinate system to space coordinate system using the operator given below.

$$\left(\frac{d}{dt}....\right)_{space} = \left(\frac{d}{dt}....\right)_{space} + \vec{\omega} \times (....)$$

$$\left(\frac{dL}{dt}\right)_{space} = \left(\frac{dL}{dt}\right)_{space} + \vec{\omega} \times \vec{L}$$

From equations 1 and 2

$$\vec{\tau} = \frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L}$$

If the body is symmetric, the axes of rotation coincides with the principal axis of symmetry.In this case, except the diagonal elements I_{xx}, I_{yy}, I_{zz}, the non-diagonal elements of the inertia tensor will be zero.

$$Let\ I_{xx} = I_1, I_{yy} = I_2, I_{zz} = I_3$$

$$\vec{\omega} \times \vec{L} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_1 & \omega_2 & \omega_3 \\ L_1 & L_2 & L_3 \end{vmatrix}$$

$$= \vec{i}(\omega_2 L_3 - \omega_3 L_2) + \vec{j}(\omega_3 L_1 - \omega_1 L_3) + \vec{k}(\omega_1 L_2 - \omega_2 L_1)$$

Hence in X direction

$$\tau_1 = \frac{dL_1}{dt} + (\omega_2 L_3 - \omega_3 L_2)$$

Since $L = I\omega$

$$\tau_1 = I_1 \frac{d\omega_1}{dt} + (\omega_2 I_3 \omega_3 - \omega_3 I_2 \omega_2)$$

$$\boxed{\tau_1 = I_1 \frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3}$$

Similarly in Y, Z directions

$$\tau_2 = I_2 \frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3$$

$$\tau_3 = I_3 \frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2$$

The above three equations are known as Euler's equations of motion of a rigid body. Expressing these equations in terms of x, y, z.

$$\tau_x = I_x \frac{d\omega_x}{dt} + (I_z - I_y)\omega_y\omega_z$$

$$\tau_y = I_y \frac{d\omega_y}{dt} + (I_x - I_z)\omega_x\omega_z$$

$$\tau_z = I_z \frac{d\omega_z}{dt} + (I_y - I_x)\omega_x\omega_y$$

Expressing these equations in symmetric form

$$\tau_x = I_x \frac{d\omega_x}{dt} - (I_y - I_z)\omega_y\omega_z$$

$$\tau_y = I_y \frac{d\omega_y}{dt} - (I_z - I_x)\omega_z\omega_x$$

$$\tau_z = I_z \frac{d\omega_z}{dt} - (I_x - I_y)\omega_x\omega_y$$

Applications of Euler's equations

Law of conservation of energy:

Euler's equations of motion are given by

$$\tau_1 = I_1\frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3$$

$$\tau_2 = I_2\frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3$$

$$\tau_3 = I_3\frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2$$

When there is no external torque acting on the rigid body $\tau = 0$

$$I_1\frac{d\omega_1}{dt} + \left(I_3 - I_2\right)\omega_2\omega_3 = 0$$

$$I_2\frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3 = 0$$

$$I_3\frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2 = 0$$

Multiplying the above equations with $\omega_1, \omega_2, \omega_3$ respectively and adding we get

$$I_1\frac{d\omega_1}{dt}\omega_1 + I_2\frac{d\omega_2}{dt}\omega_2 + I_3\frac{d\omega_3}{dt}\omega_3 = 0$$

$$\frac{1}{2}\frac{d}{dt}[I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2] = 0$$

$$I_1\omega_1^2 + I_2\omega_2^2 + I_1\omega_3^2 = K = \text{Rotational kinetic energy}$$

$$\frac{1}{2}\frac{d}{dt}(2K) = 0$$

$$\frac{dK}{dt} = 0$$

$$K = \text{Constant}$$

Hence the rotational kinetic energy of a rigid body remains constant in the absence of net external torque.

Law of conservation of angular momentum:

When there is no external torque acting on the rigid body $\tau = 0$

$$I_1\frac{d\omega_1}{dt} + \left(I_3 - I_2\right)\omega_2\omega_3 = 0$$

$$I_2\frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3 = 0$$

$$I_3\frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2 = 0$$

Multiplying the above equations with $I_1\omega_1, I_2\omega_2, I_3\omega_3$ respectively and adding we get

$$I_1^2\frac{d\omega_1}{dt}\omega_1 + I_2^2\frac{d\omega_2}{dt}\omega_2 + I_3^2\frac{d\omega_3}{dt}\omega_3 = 0$$

$$\frac{1}{2}\frac{d}{dt}[I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2] = 0$$

$$\frac{1}{2}\frac{d}{dt}\left[L^2\right] = 0$$

$$L\frac{dL}{dt} = 0$$

$$\frac{dL}{dt} = 0$$

$$L = Constant$$

Hence the angular momentum of a rigid body remains constant in the absence of net external torque.

Unit-IV
Central forces
Central force definition and examples

A force which always acts towards or away from a fixed point and whose magnitude depends only on the distance of the particle from the fixed point is known as a central force.

$$\text{Central force } \vec{F} = f(r)\hat{r}$$

Examples:

1. Gravitational force is a central force.

Gravitational force between two objects of masses m_1, m_2 separated by a distance r is given by

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

$$Let\ -Gm_1m_2 = C$$

$$\therefore \vec{F} = \frac{C}{r^2}\hat{r}$$

$$f(r) = \frac{C}{r^2}$$

$$\therefore f(r) \propto \frac{1}{r^2}$$

2. Electrostatic force is a central force.

Electrostatic force between two particles of charges q_1, q_2 separated by a distance r is given by

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{r}$$

$$Let \frac{q_1q_2}{4\pi\epsilon_0} = C$$

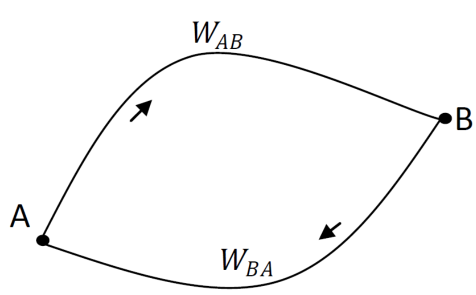
$$\therefore \vec{F} = \frac{C}{r^2}\hat{r}$$

$$f(r) = \frac{C}{r^2}$$

$$\therefore f(r) \propto \frac{1}{r^2}$$

To prove that Central force is a conservative force

If the work done by a force in moving a particle from one point to another is independent of the path followed then such force is known as a conservative force. (Or)
If the work done by a force in moving a particle around a closed path is zero then the force is known as a central force.



Work done by the central force $\vec{F}$ in moving the particle from A to B is given by

$$W_{AB} = \int_A^B \vec{F}.\vec{dr}$$

$$\text{Central force } \vec{F} = f(r)\hat{r}$$

$$W_{AB} = \int_A^B f(r)\hat{r}.\vec{dr}$$

$$\vec{r}.\vec{r} = r^2$$

Differentiating on both sides $\vec{r}.\vec{dr} + \vec{dr}.\vec{r} = 2r\,dr$

$$2\vec{r}.\vec{dr} = 2r\,dr$$

$$\vec{r}.\vec{dr} = r\,dr$$

$$\frac{\vec{r}.\vec{dr}}{r} = dr$$

$$\hat{r}.\vec{dr} = dr$$

$$\therefore W_{AB} = \int_A^B f(r)\hat{r}.\vec{dr} = \int_A^B f(r)\, dr$$

$$W_{AB} = \int_A^B f(r)\, dr$$

Value of this integral depends only on the nature of the function and the limits.

Hence $W_{BA} = \int_B^A f(r)\, dr = -\int_A^B f(r)\, dr = - W_{AB}$

$$\therefore W_{AB} + W_{BA} = 0$$

Hence the work done by central force in moving a particle around the closed path is zero.

Properties of Central forces

- ✔ A force which always acts towards or away from a fixed point and whose magnitude depends only on the distance of the particle from the fixed point is known as a central force.

Central force $\vec{F} = f(r)\hat{r}$

- ✔ Central force is a conservative force. Work done by a central force in moving a particle from one point to another is independent of the path followed.
- ✔ Under the action of a central force the torque acting on a particle is zero.
- ✔ Under the action of a central force the angular momentum of a particle remains constant.
- ✔ Under the action of a central force the aerial velocity remains constant.

Areal Velocity= $\frac{dA}{dt} = \frac{h}{2}$ =Constant

Equation of motion of a particle under the action of a central force

When the Central force acts on a particle, the acceleration is always in the direction of the radius vector. This acceleration is known as radial acceleration.

Radial acceleration $a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$

Under Central force, the transverse acceleration is always zero.

Transverse acceleration $a_t = \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 0$

$$r^2\frac{d\theta}{dt} = h = Constant$$

$$\frac{d\theta}{dt} = \frac{h}{r^2}$$

$$Let \quad r = \frac{1}{u}$$

$$\frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{h}{r^2} = -h\frac{du}{d\theta}$$

$$\frac{dr}{dt} = -h\frac{du}{d\theta}$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{d}{dt}\left(-h\frac{du}{d\theta}\right) = -h\frac{d}{dt}\left(\frac{du}{d\theta}\right) = -h\frac{d}{d\theta}\left(\frac{du}{d\theta}\right)\frac{d\theta}{dt} = -h\frac{d^2u}{d\theta^2}\frac{h}{r^2} = -h^2u^2\frac{d^2u}{d\theta^2}$$

$$\frac{d^2r}{dt^2} = -h^2u^2\frac{d^2u}{d\theta^2}$$

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -h^2u^2\frac{d^2u}{d\theta^2} - r\frac{h^2}{r^4} = -h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3$$

From Newton's Second law

$$F = -ma_r = -m\left(-h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3\right)$$

$$F = m\left(h^2u^2\frac{d^2u}{d\theta^2} + h^2u^3\right)$$

$$\frac{F}{m} = h^2u^2\left(\frac{d^2u}{d\theta^2} + u\right)$$

$$p = h^2u^2\left(\frac{d^2u}{d\theta^2} + u\right)$$

$$\frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2}$$

Kepler's first law of planetary motion

Every planet revolves around the sun in an elliptical orbit with the sun at one of its foci. This is known as Kepler's first law of planetary motion.

Let a planet of mass 'm' revolve around the sun of mass M in an elliptical orbit.

Gravitational force $F = \frac{GMm}{r^2} = \frac{\mu m}{r^2}$

$$\because GM = \mu = Constant$$

$$p = \frac{F}{m} = \frac{\mu}{r^2}$$

Equation of motion of a particle under the action of Central force

$$\frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2} = \frac{\mu}{r^2h^2u^2} = \frac{\mu}{h^2}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2}$$

$$\frac{d^2u}{d\theta^2} + \left(u - \frac{\mu}{h^2}\right) = 0$$

$$\frac{d^2}{d\theta^2}\left(u - \frac{\mu}{h^2}\right) + \left(u - \frac{\mu}{h^2}\right) = 0$$

$$let u - \frac{\mu}{h^2} = X$$

$$\frac{d^2X}{d\theta^2} + X = 0$$

Solution of this differential equation

$$X = Acos\left(\theta - \theta_0\right)$$

$$u - \frac{\mu}{h^2} = X = Acos\left(\theta - \theta_0\right)$$

$$u = \frac{\mu}{h^2} + Acos\left(\theta - \theta_0\right)$$

$$u = \frac{\mu}{h^2}\left[1 + A\frac{h^2}{\mu}cos\left(\theta - \theta_0\right)\right]$$

$$\frac{1}{r} = \frac{1+A\frac{h^2}{\mu}cos\left(\theta-\theta_0\right)}{\frac{h^2}{\mu}}$$

This equation is similar to the equation of a Conic.

$$\frac{1}{r} = \frac{1+ecos\,\theta}{l}$$

Eccentricity $e = \frac{Ah^2}{\mu}$

Kinetic Energy $K.E = \frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right]$

$$\frac{dr}{dt} =- h\frac{du}{d\theta} and \frac{d\theta}{dt} = \frac{h}{r^2}$$

$$K.E = \frac{1}{2}m\left[h^2\left(\frac{du}{d\theta}\right)^2 + r^2\frac{h^2}{r^4}\right] = \frac{1}{2}m\left[h^2\left(\frac{du}{d\theta}\right)^2 + h^2u^2\right]$$

$$K.E = \frac{1}{2}mh^2\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right]$$

$$u = \frac{\mu}{h^2} + A\cos cos\left(\theta - \theta_0\right)$$

$$\frac{du}{d\theta} =- A\sin sin\left(\theta - \theta_0\right)$$

$$K.E = \frac{1}{2}mh^2\left[A^2\left(\theta - \theta_0\right) + \frac{\mu^2}{h^4} + A^2\left(\theta - \theta_0\right) + \frac{2\mu A}{h^2}\cos cos\left(\theta - \theta_0\right)\right]$$

$$K.E = \frac{1}{2}mh^2\left[A^2 + \frac{\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos cos\left(\theta - \theta_0\right)\right]$$

Potential Energy $P.E = \int_{\infty}^{r} Fdr = \int_{\infty}^{r}\frac{\mu m}{r^2}dr = \mu m\int_{\infty}^{r}\frac{1}{r^2}dr = \mu m\left(-\frac{1}{r}\right)_{\infty}^{r} =- \mu m\left(\frac{1}{r}\right)_{\infty}^{r}$

$$P.E =- \mu m\frac{1}{r} =- \mu m u$$

$$P.E = -\mu m\left[\frac{\mu}{h^2} + A\cos cos\left(\theta - \theta_0\right)\right]$$

$$= -m\left(\mu\frac{\mu}{h^2} + \mu A\cos cos\left(\theta - \theta_0\right)\right) =- m\left(\frac{\mu^2}{h^2} + \mu A\cos cos\left(\theta - \theta_0\right)\right)$$

$$=- \frac{1}{2}mh^2\left(\frac{2}{h^2}\frac{\mu^2}{h^2} + \frac{2}{h^2}\mu A\cos cos\left(\theta - \theta_0\right)\right) =- \frac{1}{2}mh^2\left(\frac{2\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos cos\left(\theta - \theta_0\right)\right)$$

$$P.E =- \frac{1}{2}mh^2\left[\frac{2\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos cos\left(\theta - \theta_0\right)\right]$$

$$E = K.E + P.E$$

$$E = \frac{1}{2}mh^2\left[A^2 + \frac{\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos cos\left(\theta - \theta_0\right)\right] - \frac{1}{2}mh^2\left(\frac{2\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos cos\left(\theta - \theta_0\right)\right)$$

$$E = \frac{1}{2}mh^2\left[A^2 - \frac{\mu^2}{h^4}\right]$$

$$\frac{2E}{mh^2} = A^2 - \frac{\mu^2}{h^4}$$

$$A^2 = \frac{\mu^2}{h^4} + \frac{2E}{mh^2} = \frac{\mu^2}{h^4}\left(1 + \frac{2E}{mh^2}\frac{h^4}{\mu^2}\right) = \frac{\mu^2}{h^4}\left(1 + \frac{2Eh^2}{m\mu^2}\right)$$

$$A = \frac{\mu}{h^2}\sqrt{1 + \frac{2Eh^2}{m\mu^2}}$$

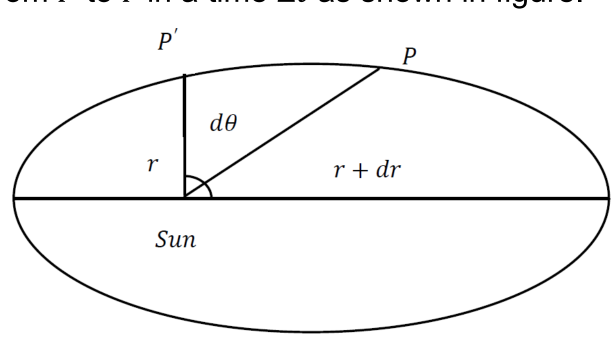
$$e = \frac{Ah^2}{\mu} = \sqrt{1 + \frac{2Eh^2}{m\mu^2}}$$

For a bound System $E < 0$. Hence Eccentricity $e < 1$.
Hence the orbit is an ellipse.

Kepler's Second law of planetary motion

The area velocity of a planet always remains constant. This is known as Kepler's Second law of planetary motion.

Consider a planet is moved from P to P' in a time Δt as shown in figure.



Area $dA = Area\ of\ the\ triangle = \frac{1}{2}r(r + dr)\sin sin\, d\theta$

$$If\,\Delta t \to 0,\ then\ \ r(r + dr) \approx r^2 and \sin sin\, d\theta = d\theta$$

$$dA = \frac{1}{2}r^2 d\theta$$

Areal Velocity$= \frac{dA}{dt} = \frac{\frac{1}{2}r^2 d\theta}{dt} = \frac{1}{2}\left(r^2\frac{d\theta}{dt}\right)$

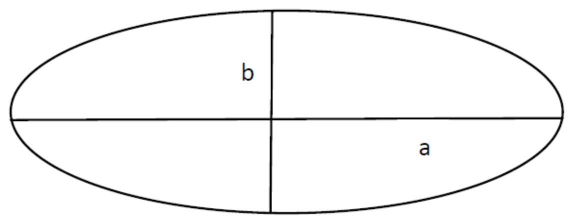
$$r^2 \frac{d\theta}{dt} = \frac{h}{2} = \text{Constant}$$

$$\text{Areal Velocity} = \frac{dA}{dt} = \frac{h}{2} = \text{Constant}$$

Kepler's third law of planetary motion

Square of the time period of a planet is directly proportional to the cube of the length of its semi-major axis

$$T^2 \propto a^3$$



Time period $T = \frac{Area\ swept\ in\ one\ revolution}{Areal\ velocity} = \frac{\pi ab}{\frac{h}{2}} = \frac{2\pi ab}{h}$

Length of semi latus rectum $l = \frac{h^2}{\mu} = \frac{b^2}{a}$

$$\frac{h^2}{\mu} = \frac{b^2}{a}$$

$$h^2 = b^2 \frac{\mu}{a}$$

$$h = b\sqrt{\frac{\mu}{a}}$$

$$\therefore T = \frac{2\pi ab}{b\sqrt{\frac{\mu}{a}}} = \frac{2\pi a\sqrt{a}}{\sqrt{\mu}}$$

$$T^2 = \frac{4\pi^2 a^3}{\mu} = \left(\frac{4\pi^2}{\mu}\right)a^3$$

$$T^2 \propto a^3$$

Unit-V
Special theory of Relativity

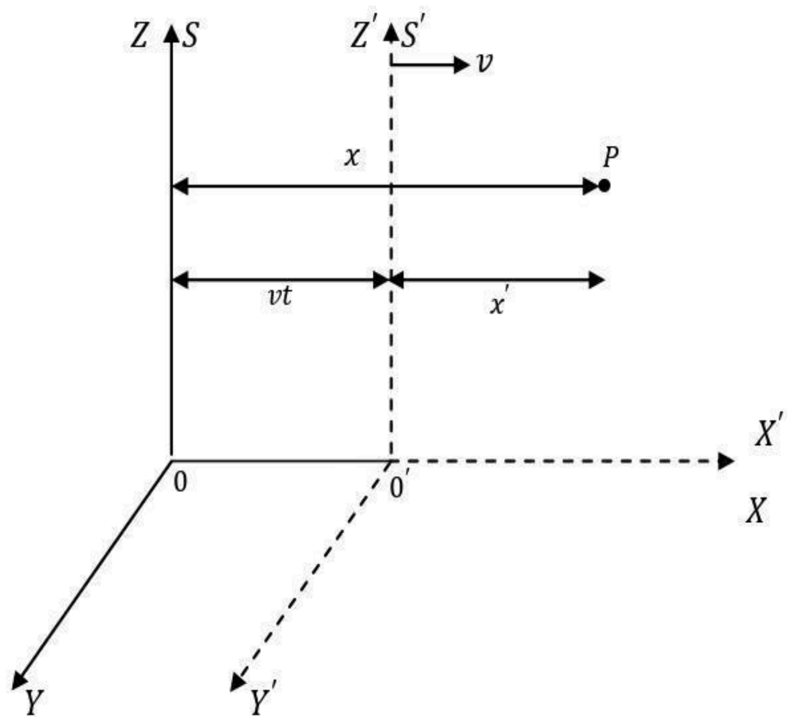
Postulates of Special theory of relativity:

- ✔ *Laws of Physics remain the same for all observers in uniform motion relative to one another*
- ✔ *Speed of light is the same for all observers in uniform motion relative to one another.*

Inertial Frame of Reference: *A frame of reference in which Newton's laws of motion are valid is known as an inertial frame of reference.*

Non-Inertial Frame of Reference: *A frame of reference in which Newton's Laws are not valid is known as a Non-inertial frame of reference.*

Galilean Transformation



Consider two inertial frames of reference S, S'. Frame S' is moving with a velocity v along the positive X-axis relative to the frame S. Let the two frames of reference S, S' coincide at time $t = 0$.

Let be the co-ordinates of the point P with respect to the frames S, S' are (x, y, z, t) and (x', y', z', t').

From Figure,

$$x = x' + vt$$
$$x' = x - vt$$

Similarly

$$y' = y,$$
$$z' = z,$$
$$t' = t$$

The above equations are called Galilean Transformation equations.

Inverse Galilean Transformation equations are

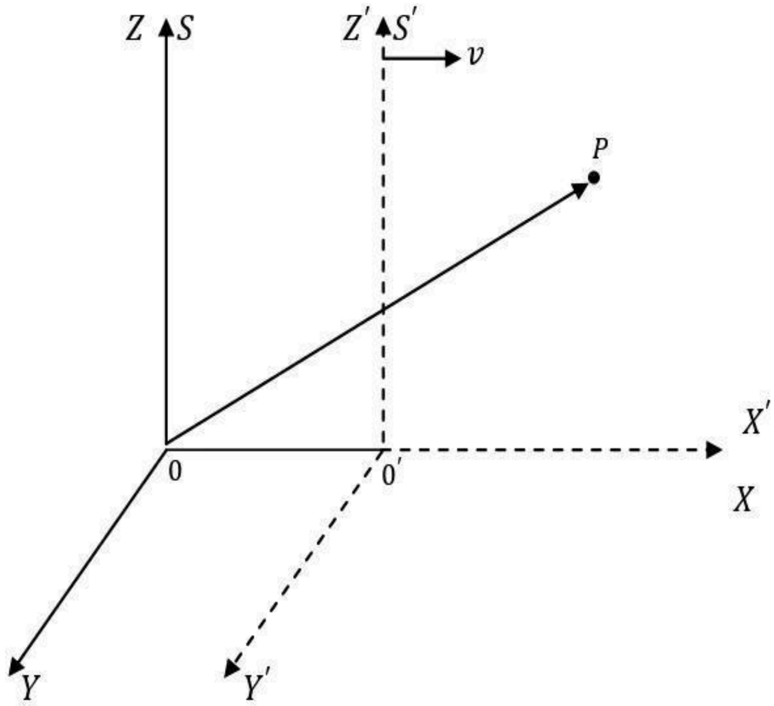
$$x = x' + vt'$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

- ➢ *Space interval is invariant under Galilean transformation*
- ➢ *Time interval is invariant under Galilean Transformation.*
- ➢ *Laws of mechanics are invariant under Galilean Transformation.*

Lorentz Transformation

Consider two inertial frames of reference S, S'. Frame S' is moving with a velocity v along the positive X-axis relative to the frame S. Let the two frames of reference S, S' coincide at time $t = 0$. Let be the co-ordinates of the point P with respect to the frames S, S' are (x, y, z, t) and (x', y', z', t').

Let a beam of light is emitted from the origin O at time $t = 0$. The beam of light reaches the point P after a time.



Relative to frame S, $$C=\frac{\text{Distance}}{\text{Time}}=\frac{\sqrt{(x^2+y^2+z^2)}}{t}$$

Relative to Frame S', $$C=\frac{\text{Distance}}{\text{Time}}=\frac{\sqrt{x'^2+y'^2+z'^2}}{t}$$

$$C=\frac{\sqrt{(x^2+y^2+z^2)}}{t}$$

$$c^2t^2=x^2+y^2+z^2$$

$$x^2+y^2+z^2-c^2t^2=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$C=\frac{\sqrt{x'^2+y'^2+z'^2}}{t'}$$

$$c^2t'^2=x'^2+y'^2+z'^2$$

$$x'^2+y'^2+z'^2-c^2t'^2=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

From equations 1 and 2

$$x^2+y^2+z^2-c^2t^2=x'^2+y'^2+z'^2-c^2t'^2$$

$$y'=y,$$

$$z'=z$$

$$x^2-c^2t^2=x'^2-c^2t'^2\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

From Galilean Transformation

$$x'=x-vt$$

$$\text{Let}\quad x'=k(x-vt)\ldots\ldots\ldots\ldots\ldots\ldots 4$$

Inverse Galilean Transformation

$$x=k(x'+vt')$$

$$x=k[k(x-vt)+vt']$$

$$\frac{x}{k}=k(x-vt)+vt'$$

$$vt'=\frac{x}{k}-k(x-vt)$$

$$vt'=\frac{x}{k}-kx+kvt$$

$$t'=\frac{x}{kv}-\frac{kx}{v}+kt$$

$$t' = kt + \left(\frac{x}{kv} - \frac{kx}{v}\right)$$
$$t' = kt - \frac{x}{v}\left(k - \frac{1}{k}\right)$$
$$t' = kt - \frac{kx}{v}\left(1 - \frac{1}{k^2}\right)$$
$$t' = k\left[t - \frac{x}{v}\left(1 - \frac{1}{k^2}\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots..5$$

From equation 3, $x^2 - c^2t^2 = x'^2 - c^2t'^2$

$$x^2 - c^2t^2 = k^2(x - vt)^2 - c^2k^2\left[t - \frac{x}{v}\left(1 - \frac{1}{k^2}\right)\right]^2$$

Comparing the coefficients of t^2 *on both sides,*

$$-c^2 = k^2v^2 - c^2k^2$$
$$c^2 = c^2k^2 - k^2v^2$$
$$c^2 = k^2(c^2 - v^2)$$
$$k^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - {v^2}/{c^2}}$$
$$k = \frac{1}{\sqrt{1 - {v^2}/{c^2}}}$$

From equation 4,

$$x' = k(x - vt)$$
$$x' = \frac{(x - vt)}{\sqrt{1 - {v^2}/{c^2}}}$$

From equation 5,

$$t' = k\left[t - \frac{x}{v}\left(1 - \frac{1}{k^2}\right)\right] = \frac{1}{\sqrt{1 - {v^2}/{c^2}}}\left[t - \frac{x}{v}\left(1 - \frac{c^2 - v^2}{c^2}\right)\right]$$
$$= \frac{1}{\sqrt{1 - {v^2}/{c^2}}}\left[t - \frac{x}{v}\left(\frac{c^2}{v^2}\right)\right] = \frac{t - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}}$$
$$t' = \frac{t - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}}$$

Lorentz Transformation Equations

$$x' = \frac{(x - vt)}{\sqrt{1 - {v^2}/{c^2}}}$$
$$y' = y,$$
$$z' = z,$$
$$t' = \frac{t - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}}$$

<u>Length Contraction or Lorentz-Fitzgerald Contraction</u>

Consider two inertial frames of reference S, S'. *Frame* S' *is moving with a velocity* v *along the positive* X*-axis relative to the frame* S. *Let the two frames of reference* S, S' *coincide at time* $t = 0$.

Let a rod of length l *is placed in the reference frame* S' *with its length parallel to* X*-axis. Co-ordinates of the ends of the rod with respect to the frames* S, S' *are* (x_1, x_2) *and* (x_1', x_2').

Time Dilation

Consider two inertial frames of reference S, S'. Frame S' is moving with a velocity v along the positive X-axis relative to the frame S. Let the two frames of reference S, S' coincide at time $t = 0$.

Let a clock be placed in the frame S.

Time interval in frame S is $\Delta t = t_2 - t_1$

Time interval in frame S' is $\Delta t' = t_2' - t_1^1$

From Lorentz Transformation

$$t' = \frac{t - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}}$$

$$\Delta t' = t_2' - t_1^1 = \frac{t_2 - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}} = \frac{t_1 - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}} = \frac{t_2 - t_1}{\sqrt{1 - {v^2}/{c^2}}} = \frac{\Delta t}{\sqrt{1 - {v^2}/{c^2}}}$$

$$\therefore \Delta t' = \frac{\Delta t}{1 - {v^2}/{c^2}} = k\Delta t$$

Case (i): When $v \ll c$ $\quad \frac{v^2}{c^2} \sim 0$

$$\therefore \Delta t' = \Delta t$$

Case(ii): When v is comparable to c

$$\Delta t' > \Delta t$$

Hence the time interval of a moving observer is more than the time interval of a stationery observer.

Case(iii): When $v = c,$ $\quad \frac{v^2}{c^2} = 1$

$$\Delta t' = \infty$$

When $v > c,$ $\quad \Delta t' =$ *Complex Number*

- ✓ *Hence no object can travel faster than the speed of light.*

Einstein's Mass-Energy Equivalence

From Newton's Second law

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

By Work-Energy theorem, work done is equal to the change in kinetic energy.

$$W = F.dx = dK$$

$$dK = F.dx = \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right)dx$$

$$= m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$$

$$= m\,dv\,\frac{dx}{dt} + v\,dm\,\frac{dx}{dt}$$

$$= m\,v\,dv + v^2 dm$$

$$\therefore dK = m\,v\,dv + v^2 dm$$

Relativistic mass

$$m = \frac{m_0}{\sqrt{1 - {v^2}/{c^2}}}$$

$$m^2 = \frac{m_0^2}{1 - v^2/_{c^2}} = \frac{m_0^2}{\frac{c^2 - v^2}{c^2}} = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$m^2(c^2 - v^2) = m_0^2 c^2$$
$$m^2c^2 - m^2v^2 = m_0^2c^2$$
$$2mc^2dm - (m^2\, 2v\, dv + v^2\, 2m\, dm) = 0$$
$$2mc^2dm = (m^2\, 2v\, dv + v^2\, 2m\, dm)$$
$$c^2dm = m\, v\, dv + v^2\, dm$$
$$dK = c^2dm$$
$$\int dK = c^2 \int_{m_0}^{m} dm$$
$$K = c^2(m)_{m_0}^{m}$$
$$K = c^2(m - m_0)$$

The above equation gives the relativistic kinetic energy of a moving body.

Energy at rest is given by

$$m_0c^2$$

Total energy

$$E = c^2(m - m_0) + m_0c^2 = mc^2$$

The above equation gives Einstein's mass-energy equivalence.

Hence Mass and Energy are not two different physical quantities. Mass can be converted in to energy and vice-versa.

Addition of Velocities or Transformation of Velocities

Consider two inertial frames of reference S, S' . Frame S' is moving with a velocity v along the positive X-axis relative to the frame S. Let the two frames of reference S, S' coincide at time $t = 0$.

In reference frame S, an object moves a distance dx in time dt. Similarly in reference frame S' , the object moves a distance dx' in time dt'.

Velocity in Reference frame S $\quad u = \frac{dx}{dt}$

Velocity in Reference frame S' $\quad u' = \frac{dx'}{dt'}$

From Lorentz Transformation

$$x' = k(x - vt)$$
$$t' = k\left(t - {vx}/_{c^2}\right)$$

From Inverse Lorentz Transformation

$$x = k(x' + vt'), t = k\left(t' + {vx'}/_{c^2}\right)$$
$$dx = k(dx' + v\, dt'), dt = k\left(dt' + {vdx'}/_{c^2}\right)$$
$$u = \frac{dx}{dt} = \frac{k(dx' + v\, dt')}{k\left(dt' + {vdx'}/_{c^2}\right)} = \frac{(dx' + v\, dt')}{\left(dt' + {vdx'}/_{c^2}\right)}$$
$$= \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$
$$u = \frac{u' + v}{1 + {u'v}/_{c^2}}$$

The above equation represents the relativistic law of addition of velocities.

Case(i) : When $u' \ll c$, $v \ll c$

$$\frac{u'v}{c^2} \sim 0$$

$$\therefore u = u' + v$$

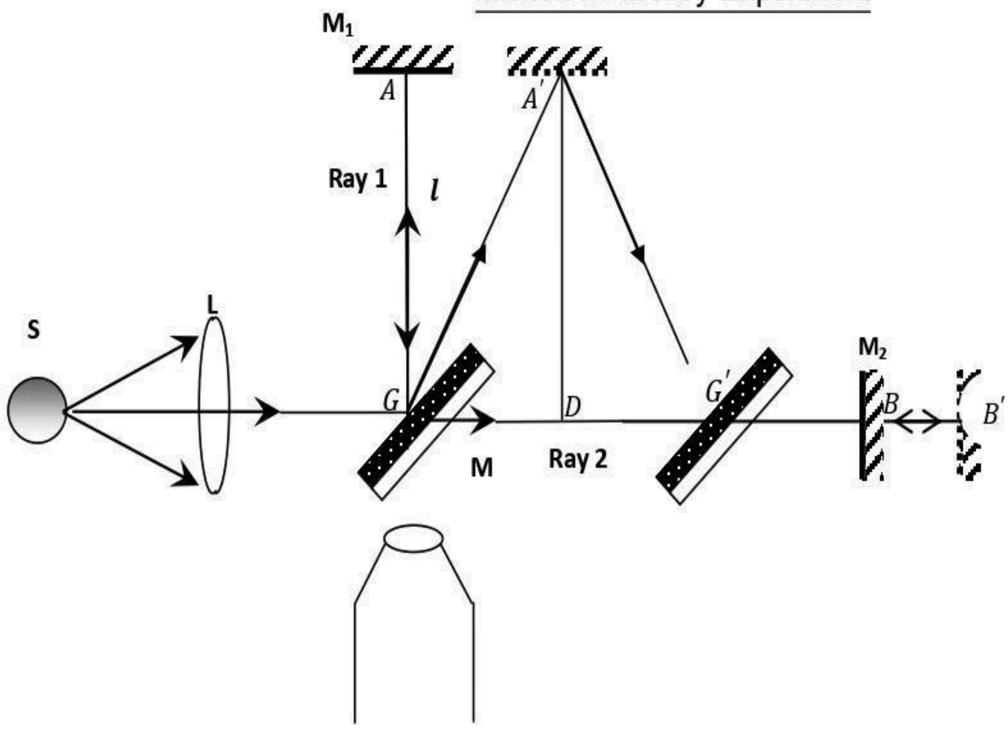
Case(ii): When $u' = c\,, v = c$

$$u = \frac{c + c}{1 + {c.c}/{c^2}} = \frac{2c}{2} = c$$

$$u = c$$

Hence addition of velocity of to the velocity of light reproduces the velocity of light.

Michelson-Morley Experiment



Aim: Aim of Michelson-Morley experiment is to determine the velocity of Earth relative to Ether.

Michelson-Morley Interferometer is shown in figure. Light emitted from the monochromatic source S *falls on the half silvered glass plate* G. *The glass plate* G *is oriented at an angle of* 45^0 *to the incident light. Hence the light incident on the glass plate* G *is divided in to two perpendicular beams of light. The two beams of light are reflected back from the two mirrors* M_1, M_2 *and meet at* G *to produce interference pattern. The interference pattern can be observed through the telescope* T.

Since the apparatus is moving with a velocity v *along with the Earth, the optical paths of two beams are not equal. The two beams are reflected at the points* A', B' *instead of* A, B *and interfere at* G'.

From $\Delta\, GA'D$ $(GA')^2 = (A'D)^2 + (GD)^2$

$$c^2t^2 = l^2 + v^2t^2$$

$$l^2 = (c^2 - v^2)t^2$$

$$t^2 = \frac{l^2}{(c^2 - v^2)}$$

$$t = \frac{l}{\sqrt{c^2 - v^2}} = \frac{l}{c\sqrt{1 - {v^2}/{c^2}}}$$

$$= \frac{l}{c}\left(1 - {v^2}/{c^2}\right)^{-1/2} = \frac{l}{c}\left(1 + {v^2}/{2c^2}\right)$$

Hence the time taken by the light beam 1 to reach G'

$$t_1 = 2t = \frac{2l}{c}\left(1 + {v^2}/{2c^2}\right)$$

Let be t_2 *the time taken by the light beam 2 to reach the glass plate* G'.

Velocity of light beam from G *to* B' *is* $(c - v)$ *and from* B' *to* G' *is* $(c + v)$.

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v} = l\left(\frac{1}{c-v} + \frac{1}{c+v}\right) = l\left(\frac{2c}{c^2 - v^2}\right)$$

$$\frac{l(2c)}{c^2\left(1 - {v^2}/{c^2}\right)} = \frac{2l}{c}\left(1 - {v^2}/{c^2}\right)^{-1} = \frac{2l}{c}\left(1 + {v^2}/{c^2}\right)$$

$$t_2 = \frac{2l}{c}\left(1 + {v^2}/{c^2}\right)$$

Time lag between the two beams

$$\Delta t = t_2 - t_1$$

$$= \frac{2l}{c}\left(1 + {v^2}/{c^2}\right) - \frac{2l}{c}\left(1 + {v^2}/{2c^2}\right)$$

$$= \frac{2l}{c}\left(1 + {v^2}/{c^2} - 1 - {v^2}/{2c^2}\right)$$

$$= \frac{2l}{c}\left(\frac{v^2}{2c^2}\right)$$

$$= \frac{lv^2}{c^3}$$

$$\Delta t = \frac{lv^2}{c^3}$$

$$\textit{Path difference} = c.\Delta t = c.\frac{lv^2}{c^3} = \frac{lv^2}{c^2}$$

Path difference in terms of Wavelength $= \frac{lv^2}{\lambda c^2}$

Mirrors M_1, M_2 *are interchanged by rotating the apparatus by* 90^0

Path difference $= -\frac{lv^2}{\lambda c^2}$

Resultant Path difference $= \frac{lv^2}{\lambda c^2} - \left(-\frac{lv^2}{\lambda c^2}\right) = \frac{2lv^2}{\lambda c^2}$

Hence Fringe Shift $\Delta n = \frac{2lv^2}{\lambda c^2}$

In Michelson-Morley Experiment, $l = 10m, v = 3 \times 10^4\ m/s\ , \lambda = 5000\ \times 10^{-10} m, c = 3\ \times 10^8\ m/s$

$$\therefore \Delta n = \frac{2 \times 10 \times (3 \times 10^4)^2}{5000 \times 10^{-10} \times (3 \times 10^8)^2} = 0.4$$

Hence a fringe shift of 0.4 *was expected. But Michelson-Morley observed a fringe shift of only 0.001. This is known as Null Result.*

Significance of Null Result:

- *It is impossible to measure the speed of Earth relative to Ether. Hence the concept of Ether is rejected.*
- *Speed of light in vacuum is the same for all observers.*